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TRANSPORTATION PROBLEM - STANDARD VS. NETWORK LINEAR PROGRAMMING: 2. THE PRIMER

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Summary. This paper presents continued general discussion introduced in [1] on two methods for solving transportation problems: standard and network linear programming. Particular attention here is put on modeling and computing issues related to both methods. Simple illustrative example is used to demonstrate how transportation problem may be attacked by two related solvers, Simplex and Out-of-kilter. Specific notes are given on knowledge analyst has to be armed with in order to be able to apply the network approach and use network methods and solvers.

1. INTRODUCTION

This paper comprises experience gained through the application of two different methodologies for solving generalized closed transportation problem (TP). The first one is based on standard linear programming and use of well-known Simplex solver [2]. Hereafter this will be denoted as StdLP approach. The second methodology is based on network integer linear programming (NetLP approach) and Ford-Fulkerson's Out-of-kilter algorithm [3]. Both solvers were appropriately programmed in FORTRAN 77. Related computer codes, entitled SX and OK respectively, are of the core type and implemented at Pentium 133 MHz computer. Core type here means that except true executive statements, which perform basic algorithms' computations, all other statements are completely deleted to fasten codes' compilation, linking and execution. This way programming proficiencies were reduced as much as possible, even some controversies may still exist.

For the comparison purposes closed TP is selected and intently created so to be enough simple but to include typical transport situation with transient flows (via transferring nodes). The TP general formulation and notation is introduced in [1], and mathematical relations given hereafter are in full consistence with it. Methodological steps performed during both solving procedures (StdLP/Simplex and NetLP/Out-of-kilter) are briefly described in turn.

2. TP - THE PRIMER

The typical generalized closed TP with 10 links and 6 nodes is shown on Fig. 1. In order to create similar starting position for solving the problem via StdLP and NetLP, it may be assumed that all nodes are both source and sink nodes. This obviously fits to either approach: if node is not really a source, it has a zero capacity; and if node is not a sink, its demand is zero. For given problem, out of 6 nodes, three nodes are only sources (1, 2 and 4), one is only a sink (6) and two are in the same time sources and sinks (3 and 5). Nodes 3 and 5 are true transferring nodes. Source and sink capacities for nodes are represented by numbers in [,] parenthesis, respectively. Node characteristics summarized in Table 1 also indicate that total source capacity of 600 volumetric units of resource is equal to total sink capacity. The problem is closed, i.e. there will be neither surpluses nor deficits in the network.

As far transporting links are concerned, they are identified by arrows, unit costs and amounts of flowing resource, Fig. 1. For example, link that connects nodes 1 and 4 (with direction from node 1 to node 4) has unit cost equal to 4. For a convenience, unit costs are indicated as numbers written near the origin of each particular link. Terms x_1, x_2, x_{10} denote volume of a resource flowing through links, and subscripts are the link identification numbers. For example, flow through the link 8 is x_8 and unit cost of this flow is 7; in this link flow may be maintained only from node 3 toward node 6. Links' characteristics are given in Tab. 2.

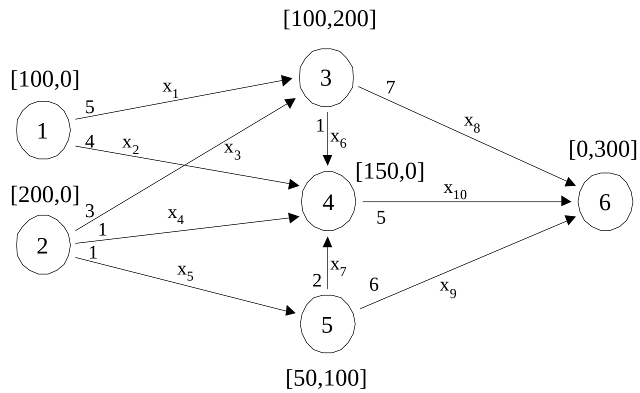

Fig. 1 Generalized closed TP: The Primer

Table 1. Generalized closed TP: Nodes

Node (j)	**Source capacity (s_j*)**	**Sink capacity (d_j*)**
1	100	0
2	200	0
3	100	200
4	150	0
5	50	100
6	0	300
Total	600	600

Table 2. Generalized closed TP: Links

Link no.	**From node (i)**	**To node (j)**	**Unit flow cost (c_{ij})**
1	1	3	5
2	1	4	4
3	2	3	3
4	2	4	1
5	2	5	1
6	3	4	1
7	5	4	2
8	3	6	7
9	5	6	6
10	4	6	5

<u>The problem to be solved is</u>: Find the set of values (x_1, x_2, x_{10}) which gives minimum total cost of distributing 600 units from all source to all sink points according to nodes' capacities, links' layouts and given unit costs of transport. Assume unrestricted links' upper limits and lower limits set to zero.

2.1. Solution via StdLP

Following the unique notation given on Fig. 1, due to model (6)-(8) given in [1] the linear program to be solved is:

$$\text{Find: } F^* = \min\,(5x_1+4x_2+3x_3+x_4+x_5+x_6+2x_7+7x_8+6x_9+5x_{10}) \qquad \ldots(1)$$

with balance conditions (at nodes):

$$\begin{array}{l}
0 + 100 = x_1 + x_2 + 0\\
0 + 200 = x_3 + x_4 + x_5 + 0\\
x_1 + x_3 + 100 = x_6 + x_8 + 200\\
x_2 + x_4 + x_6 + x_7 + 150 = x_{10} + 0\\
x_5 + 50 = x_7 + x_9 + 100\\
x_8 + x_9 + x_{10} + 0 = 300
\end{array} \qquad \ldots(2)$$

and constraints (on links):

$$x_1, x_2, \ldots, x_{10} \geq 0\,. \qquad \ldots(3)$$

Relation (9) in [1] is clarified by total node capacities in Table 1. Rewriting relations (1)-(3) and adding artificial variables puts the original LP into the standard form:

$$\begin{array}{l}
\text{Find: } F^*=\min\,[5x_1+4x_2+3x_3+x_4+x_5+x_6+2x_7+7x_8+6x_9+5x_{10}+\\
\qquad\qquad +M(x_{11}+x_{12}+x_{13}+x_{14}+x_{15}+x_{16})]
\end{array} \qquad \ldots(4)$$

with balance conditions (at nodes):

$$\begin{array}{cccccccccccccccccccccccc}
x_1 & + & x_2 & & & & & & & & & & & & & & & & & + & x_{11} & = & 100\\
 & & & & x_3 & + & x_4 & + & x_5 & & & & & & & & & & & + & x_{12} & = & 200\\
x_1 & & & + & x_3 & & & & & - & x_6 & & & - & x_8 & & & & & + & x_{13} & = & 100\\
 & & x_2 & & & + & x_4 & & & + & x_6 & + & x_7 & & & & & - & x_{10} & + & x_{14} & = & -150\\
 & & & & & & & & x_5 & & & - & x_7 & & & - & x_9 & & & + & x_{15} & = & 50\\
 & & & & & & & & & & & & & & x_8 & + & x_9 & + & x_{10} & + & x_{16} & = & 300
\end{array} \qquad \ldots(5)$$

and constraints (on links):

$$x_1, x_2, \ldots, x_{16} \geq 0 \qquad \ldots(6)$$

where $x_{11}, \ldots, x_{16}$ are artificial variables and M is arbitrarily selected positive large number.

Standard linear program (4)-(6) may readily be solved by Simplex. After 6 steps algorithm produces unique optimal solution: $F^* = F_{min} = 2200$; $x_1^*=100$, $x_4^* =150$, $x_5^*=50$ and $x_{10}^*=300$; all other variables equal to zero.

2.2. Solution via NetLP

To obtain network linear program for TP on Fig. 1 there is no need to write any equation. Due to modeling strategy described in [1] it is enough to create an oriented closed capacitated network and consider it as The Minimal Cost Flow Problem. In given example original network shown on Fig. 1 has to be supplemented by few additional nodes and links in such a manner to become closed. Resulting network may be denoted as artificial network, Fig. 2. It's topology and parameters will be described in turn.

The new network consists of two parts: (1) original network with 6 nodes and 10 links, and (2) additional network with 3 nodes and 10 links. The logic of including new nodes and links is described in details in [1], and the following comments serve to explain 'the network way of thinking'.

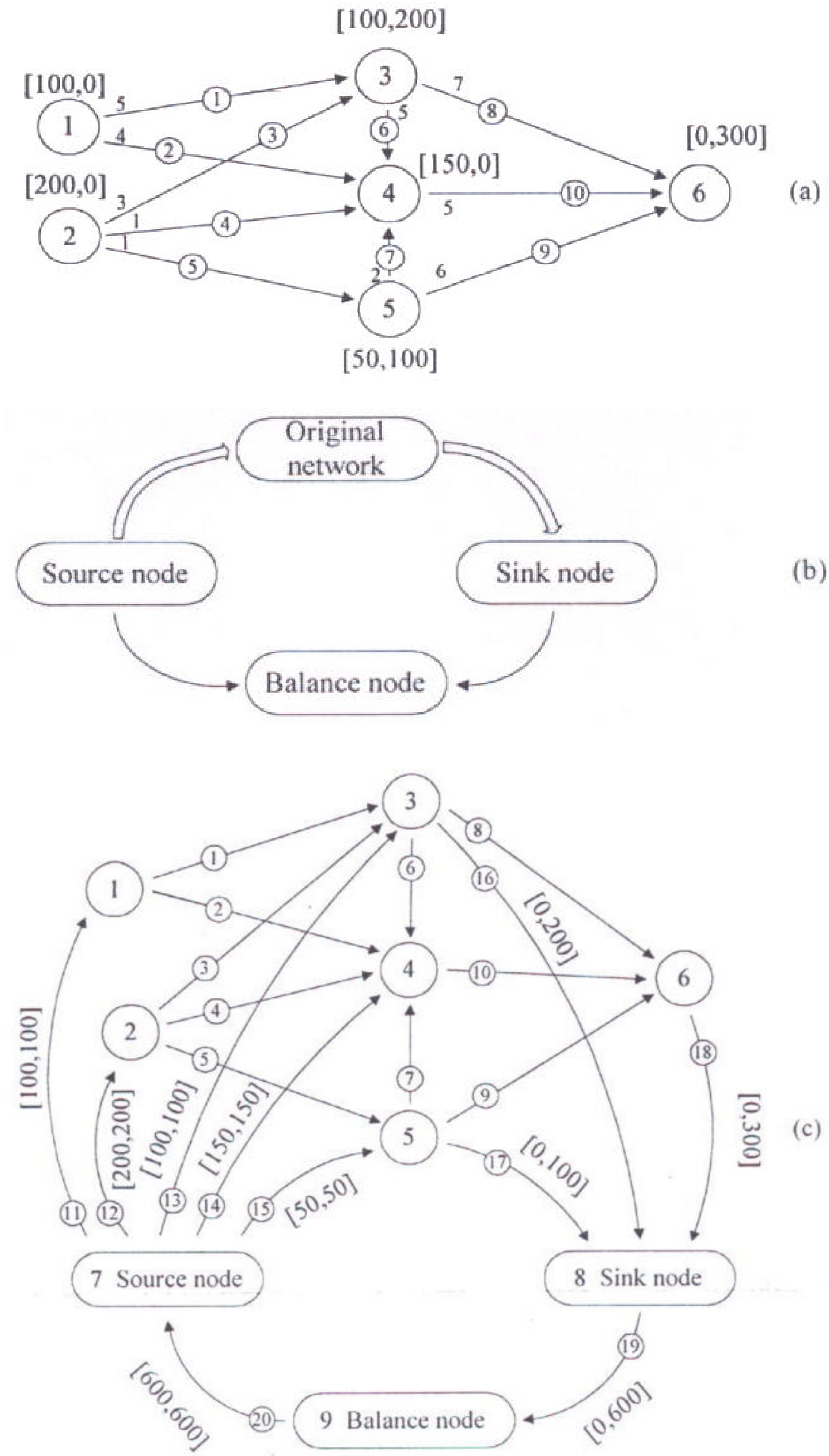

Fig. 2 NetLP formulation of an example TP (clarify Fig. 1)

Let analyze the situation at node 5 being a source and sink node at the same time. Two original links (numbered 7 and 9) leave this node, one link comes in from node 2 (numbered 5) and one additional link (numbered 15) comes in from node denoted as 'Source node' (numbered 7). Additional link simulates source capacity at node 5. Having equal lower and upper limit (here 50 as specified in [] parenthesis), this link during solving procedure will permit transfer of exactly 50 units of resource. This means that optimal solution will for sure include optimal value $x_{15}*=50$. Since 50 units must come to node 5 because link 15 dictates it, node 5 will have at least 50 available resource units for further distribution (through links 7, 9 and 17). Note that term 'at least 50' here means that certain amount of resource may also come to node 5 through link 5 forming thus available amount of resource greater than 50.

The link 17 belongs to the set of additional links. It leaves node 5 providing flow to additional node denoted as Sink node 8. Specified flow limits on this link [0,100] in fact model sink capacity of node 5. Lower limit is set to 0 to permit possible situation that sink capacity (in fact demand at node 5) will not be fulfilled to the maximum of 100. This setting is intently used to indicate modeling strategy when open generalized TP should be solved; in this case shortages at demand point 5 could occur. Of course, to provide sure satisfaction of demand at point 5, limits on link 17 should be equaled to 100, i.e. [100,100] should be specified instead of [0,100].

To summarize situation at node 5, two links model available resource at node for further distribution (links 15 and 5) and three links model the usage of available resource at node (link 17 provide satisfaction of demand located at node, and links 7 and 9 transfer the rest of resource to nodes 4 and 6).

Generally, the set of initial source capacity links connecting Source node 7 with all original nodes should be specified with appropriate limits analog to the one described for node 5. However, since node 6 has source capacity equal to zero, related link is avoided; it could be included with limits [0,0], which have no sense because flow through that link could not exist. The same logic applies for Sink node 8. At the most, six links should enter this node leaving set of original nodes. Here, three nodes (1, 2 and 4) have not specified demands (sink capacities) so related links are simply avoided.

The link 19 represents total sink capacity of the network and its limits are specified as sums of respective limits on links entering Sink node 8. It may bring at maximum 600 units of resource to the Balance node 9. The link 20 represents total source capacity of all original nodes. It's min/max flow limits are specified as sums of respective limits on links leaving Source node 7. Herein limits are [600,600] indicating that link 20 will transfer exactly 600 units of resource to Source node 7; at that point it will be divided into exact amounts and transferred through arcs 11-15 to appropriate original nodes (except node 6 where there is not source capacity).

The role of additional nodes (7-9) and links (11-20) is obvious. They model node conditions with respect to their capacities and completely close initial network. It should be noted that nodes 7-9 have to be in balance (like all other nodes), which may be achieved only if all available source resource is sinking (used).

To summarize, flows in additional network shall be maintained only as follows:

- Links on source side of the network: $x_{11}=100$, $x_{12}=200$, $x_{13}=100$, $x_{14}=150$, $x_{15}=50$, and $x_{20}=600$.
- Links on sink side of the network: $x_{16}=200$, $x_{17}=100$, $x_{18}=300$, and $x_{19}=600$.

These flows are in fact prespecified and make the part of final, optimal solution.

As far unit costs are considered, it is best to specify unit costs equal to zero for additional links 11-20 in order to eliminate their influence on extended criterion function. For given TP this is rational approach since in additional links flows should be maintained no matter what unit costs are. However, in open TP where deficit or sufficit of resource may occur, it could be opportune to define different (and even negative) unit costs for these links due to logic of preferencing certain demands in original network (see [1] for details). In this way it is possible to preserve all information contained in priority matrix of demand nodes.

The Out-of-kilter algorithm applied to closed capacitated network depicted on Fig. 2c produces the same optimal solution as Simplex algorithm under StdLP procedure.

3. COMPUTATIONAL ISSUES

To evaluate and compare selected procedures (StdLP vs. NetLP) and related algorithms (Simplex vs. Out-of-kilter), two original computer programs (SX and OK, respectively) were used. Written in FORTRAN 77 both programs were adapted for testing purposes only by extraction of all unnecessary statements. Special return codes were installed in both core programs at starting points of true computations. For SX it is the statement where algorithm seeks for 'basis' variables; for OK it is the statement where labeling procedure starts. End of computation return codes were installed in both programs appropriately. In this way start/end switches in SX bounded exactly solving procedure, i.e. iterative process of generating simplex tables. Iterative process of balancing nodes (i.e. putting them into so called in-kilter status /4/) in OK was identified by similar two switches. To put both programs into same testing environment, manual data preparation is performed although original programs posses very sophisticated and user-friendly software solutions.

Table 3 summarizes principal characteristics of both 'core version' programs. Computing times are given for extremely simple generalized closed TP on Fig. 1 and 2c and therefore are provisional. However, original SX and OK were used for solving large-scale closed and opened TP of sizes up to 200 nodes and 1000 links. All computer works were performed on Pentium 133MHz/32 MB RAM. Generally, OK is 4 times faster than SX.

Table 3. Computer programs characteristics

Characteristics	**SX program (Simplex)**	**OK program (Out-of-kilter)**
Language	FORTRAN 77	FORTRAN 77
No. of source code lines	76	214
Memory occupation	34.4 KB	43.5 KB
Program structure	Main program only	Main p. + SUPERK routine /4/
Max. problem dimension	Max. 30 LP variables	200 arcs and 36 nodes in network
Computing time*	0.03 sec	0.01 sec

*Computing time could be measured only with 0.01 sec precision.

4. CONCLUSIONS

The handling (modeling and solving) of transportation problems in operations research and systems analysis practice is usually faced with certain and important dilemmas. Probably the most important one is 'Which modeling strategy to choose?'. In order to stipulate discussion in this direction two alternatives are presented and evaluated: (1) Standard LP with Simplex algorithm as solver, and (2) Network (Integer) LP with Out-of-kilter solver. In latter case, appropriate transformation of the original TP into The Minimal Cost Flow Problem is assumed.

Although selection of one among two offered alternatives is faced with many subjective judgments, recent investigations indicate greater flexibility of network LP approach over standard LP approach, particularly if large scale and/or dynamical TPs have to be solved [5,6,7]. As far hardware and software requirements are concerned, standard LP is still prevailing in practice, while network LP is more commercialized. An important reason why network modeling is still not so popular is that it is at some instances 'harder' than standard modeling. The other reason seems to be recognizable lack of educated professionals to ensure effective promotion of this modern operational research approach in different areas of human activities.

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