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TRANSPORTATION PROBLEM - STANDARD VS NETWORK LINEAR PROGRAMMING: 1. GENERAL SCOPE

Bojan Srdjevic
Institute for Water Management, Faculty of Agriculture, University of Novi Sad
2100 Novi Sad, Trg D. Obradovica 8, Yugoslavia
E-mail: bojans@polj.ns.ac.yu

Summary. Probably the most important and interesting in operational research and systems analysis is the class of transportation problems (TPs). Large scale and particularly dynamical TPs, closed and opened, need very careful analysis before appropriate salvation procedure and method should be selected. Acting in this course, discussion in this paper is devoted to general characteristics and issues related to implementation of two possible procedures: standard linear programming and network linear programming. Assuming the use of two well known and popular LP solvers, simplex and out-of-kilter, consistent with either of those two approaches, discussion puts a stress on modeling issues before algorithms should effectively be applied. Advantages and disadvantages of two LP approaches and solvers are also identified in order to help eliminating subjective judgments on which one to select for practical use in solving transportation problems.

1. INTRODUCTION

Transportation problem (TP), particularly large scale TP, implies exhaustive solving procedure even sophisticated software and high-speed computer are used. Stating the TP appears to be a serious problem itself, either 'static' or 'dynamic' one is considered. If this initial phase is successfully resolved, solution methodology is generally straightforward. However, final part of procedure - preparation and presentation of obtained results to decision makers - may become very critical. Due to political, economical and other possible influences, compromises and deviations from optimal solution become more and more normal outcome. It is not therefore surprising that anticipated frustration of the analyst is built-in within all steps of above mentioned procedure. Preparedness for additional investigations of sub optimal solutions or even restating initial TP and repeating the whole solving procedure becomes an important issue the analyst is faced with.

As far dynamic, i.e. time dependent, TPs are concerned, recent investigations indicate that true optimization is almost impossible. Main reason is that internal model structure becomes very complicate due to difficulties in modeling transient effects when time period is discretized into given number of time frames. Additional complications arise when dynamic transportation process possesses stochastic nature. Stochastic inputs do not represent part of internal model structure but rather imply in turn relaxation of optimal procedure no matter how precisely it is defined. An important fact is that, by definition, mathematically rigorous optimum may not be attained. As a consequence, switch to mixed simulation/optimization is often used as rational judgment and approach, which furthermore may lead to significant difficulties whenever dynamic TP had to be modeled and solved lately.

To select linear programming technique to solve given TP, i.e. to minimize total cost of flows within a network while satisfying prescribed restrictions, is obviously an important decision of the system analyst. Two questions would be answered first to avoid latter difficulties. The first one is: how the logic of TP fits with different available linear programming algorithms? For example, what should be the better - standard linear programming algorithm, or special TP algorithm, or maybe special linear programming network algorithm? Second question is: has an analyst in hand any appropriate software, and of what type is it? The latter one is closely related to available hardware, which generally means use of mainframe or PC.

This paper discusses the important issues related to solving generalized TP (usually referred in literature as Transshipment Problem), via two different linear programming (LP) approaches and solvers: 1. Standard LP (hereafter denoted as StdLP), and 2. Network LP (NetLP). Related solvers are well known Simplex, which belongs to the class of standard LP algorithms [1], and Ford-Fulkerson's Out-of-Kilter

[2] from the class of network LP algorithms. Details on those two well-known algorithms may be found in reach pertinent literature and therefore are completely avoided here.

Descriptive and mathematical statement of classical and generalized TP is given in chapters 2 and 3 due to LP standards, i.e. anticipating use of Simplex algorithm. Chapter 4 presents basic characteristics of network approach to TP avoiding modeling details. Since network modeling strategy itself comprises broad scope of special concepts, mathematical tools, procedures and software, it's brief overview in chapter 4 is given to serve as bridge toward commonly used standard LP strategies. Advantages and disadvantages of two selected solving procedures (via Simplex and Out-of-Kilter) are presented in chapter 5 and conclusions in the last one.

2. CLASSICAL TP

Classical (closed) TP, Fig. 1, may shortly be stated as follows: Transport of resource is to be maintained from given number of 'source nodes' where constant volume of resource is available. Available resource should be transported directly to given number of 'sink nodes' where constant volume of resource is needed. Transport should be maintained between source nodes and sink nodes along specified links, assuming that it is not necessary to have links connecting each source with each sink. Total volume of resource in all sources is equal to volume of resource demanded at all sinks. For each transportation link the unit transport cost is specified. Total cost of transport is defined as linear function, i.e. cost of transport along given link is proportional to an amount of resource transported. Total cost of transport in a network is a sum of costs of resource transported along each link.

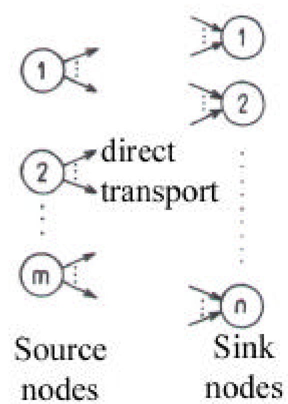

Fig. 1 Classical Transportation Problem

Transportation problem may also be formulated as minimal cost flow problem. In other words, the problem is to find minimal total cost of resource transport from source to sink nodes satisfying above stated conditions and constraints.

The mathematical formulation of <u>classical closed TP</u> consists of few simple relations:

Find:

$$F^* = \min \left(F = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \right) \quad \ldots(1)$$

under given constraints:

$$\sum_{j=1}^{n} x_{ij} = S_i, \qquad i=1,\ldots,m \quad \ldots(2)$$

$$\sum_{i=1}^{m} x_{ij} = D_j, \qquad j=1,\ldots,n \quad \ldots(3)$$

$$x_{ij} \geq 0, \qquad i=1,\ldots,m; \; j=1,\ldots,n \quad \ldots(4)$$

and condition:

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j \ . \qquad \ldots (5)$$

Total number of source and sink nodes is denoted by m and n, respectively. S_i is amount of available resource at source node i, and D_j is amount of demanded resource at sink node j. x_{ij} represents amount of resource transported along link (i,j), while c_{ij} is unit cost of transport. Finally, F is linear objective function (total cost of transport within a network), and F* is it's optimal value.

For classical open TP, condition (5) changes to

$$\sum_{i=1}^{m} S_i \neq \sum_{j=1}^{n} D_j \qquad \ldots (5')$$

reflecting a situation when total available resource must not be equal to total demand.

To solve open TP there are different ways. Advisable one is to simply introduce one artificial source or sink node 'to give a way for resource surplus or deficit to flow outside the original network'. The addition of artificial node and appropriate links immediately turns the open TP back to closed TP represented by (1)-(5), of course now with slightly expanded sets of nodes and links. Some interpretation of obtained result is the only difficulty left to analyst at the end of solving procedure.

Recent TP statement does not include upper constraints for links. Since maximum link capacities are not defined, they may be assumed as sufficiently large. On the contrary, minimum capacities are specified as zeroes. Inclusion of maximum capacity constraints for links should incur expansion of above stated linear program (1)-(5) in part of constraints. In later program standardization this should lead to introduction of one additional variable per link and expansion of objective function appropriately.

3. GENERALIZED TP

Generalized transportation problem differs from classical one in three important characteristics. The first one is that resource generally must not be directly transported from sources to sinks, i.e. on the 'ways' from sources to sinks there may exist some 'transferring' nodes. The second difference is that each node in a network may be only a source node, only a sink node or may be both in the same time. Finally, each node must be in balance, which means that all inputs into a node must be balanced (equaled) with outputs from node. This constraint should be understood as follows: amount of resource available at node (source of resource) plus resource transported from other nodes to that node must be equal to an amount of resource to be used at that node (sink of resource) plus resource transported to other nodes. Fig. 2 gives an illustration of generalized TP.

Assuming all other statements given in previous chapter are still valid, and all nodes are numbered uniquely, mathematical formulation of generalized closed TP is as follows:

Find:

$$F^* = \min \left(F = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \, x_{ij} \right) \qquad \ldots (6)$$

with balance conditions (at nodes):

$$\sum_{i \neq j} x_{ij} + s_j^* = \sum_{k \neq j} x_{jk} + d_j^* = v_j^+ , \qquad j=1,\ldots,n \qquad \ldots (7)$$

constraints (on links):

$$x_{ij} \geq 0 , \qquad i=1,\ldots,n; \ \ j=1,\ldots,n \quad (i \neq j) \qquad \ldots (8)$$

and condition:

$$\sum_{i=1}^{n} S_i = \sum_{i=1}^{n} D_i \qquad \ldots (9)$$

Here, n is total number of nodes in a network, s_j^* is 'capacity' (source) of a node j, b_j^* is demand (sink) at node j, and v_j^+ is amount of resource which may be consumed at node j or transported to other nodes.

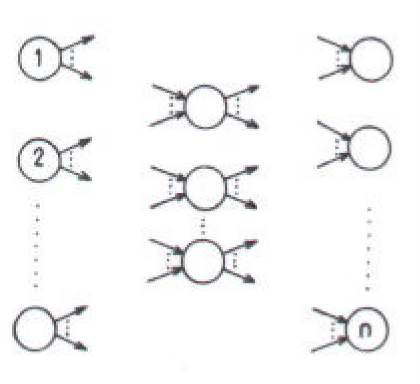

Fig. 2 Generalized Transportation Problem

Similar to classical open TP, total 'source capacity' in generalized open TP may not be equal to total 'sink capacity':

$$\sum_{i=1}^{n} S_i \neq \sum_{i=1}^{n} D_i \qquad \ldots (9')$$

which assumes similar additions of artificial nodes and links and solving procedure like in previous chapter.

Models (6)-(9) and (6)-(9') correspond to **StdLP** formulation and usually are termed as non-standard (non-canonical) linear programs. To solve either one by Simplex it is necessary to transform program into appropriate standard (canonical) form by adding slack, surplus or artificial variables.

4. NETWORK APPROACH TO TP

Recent investigations indicate that all varieties of TPs may directly be solved as minimal cost flow problems by use of oriented closed capacitated networks and related network solvers *(algorithms)*. To link network terminology with the one used in previous chapters, few notes should be given related to oriented closed capacitated network topology and parameters.

4.1. Network

A network is set of points, called nodes, and set of curves, called arcs, which connect certain pairs of nodes. An example network on Fig. 3 consists of five nodes, labeled 1 through 5, and six arcs: (1,2), (1,3), (2,3), (1,4), (3,4) and (4,5). An arc is *oriented* if it has a direction associated with it. Schematically, arrows indicate directions. The arrow on arc (1,2) in Fig. 3 signifies that this arc is directed from node 1 to node 2. Any movement along this arc must originate at node 1 and terminate at node 2. Movement from node 2 to node 1 is not permitted along that arc; such a movement should be possible if the other arc exists between those two nodes with opposite direction. If all arcs are directed, the *network is oriented.*

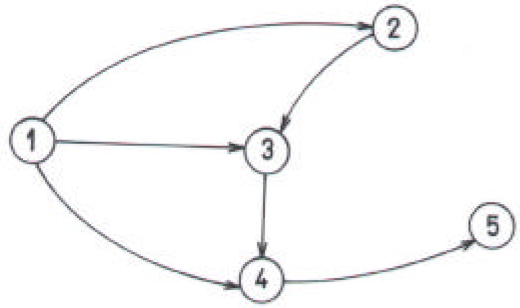

Fig. 3 An example network

4.2. Oriented closed capacitated network

Slightly different notation shall be used to simplify further exposition of a problem. Pair of numbers in parenthesis (ij) identifies an arc in a network connecting nodes i and j with the direction from i to j. Set of two or more arcs (neglecting their directions), which connect any pair of nodes p, and q makes a *path* between those nodes. If p = q, i.e. starting node is also a terminal node, a path is *closed*. Network is *connected* if there is a path between any two nodes. When all arcs along a path are so oriented that it is possible to transport unit of flow from node p to node q, the *path is oriented*. If p = q, such path is *oriented closed path* and such a network is called *oriented closed network*.

Flow through a network may be understood as stationary movement of given homogeneous resource or commodity (gas, water, information, fruit...) along network arcs. Volume of resource flowing through an arc (ij) is usually denoted as x_{ij}, while symbols u_{ij} and l_{ij} specify max/min limits on this flow. When each arc has specified flow limits, such a network is termed *oriented closed capacitated network*. Additional feature that for each arc unit cost of flow, c_{ij}, may be specified, gives basis for approaching any TP as Minimal Cost Flow Problem.

4.3. Mathematical NetLP formulation

Linear program for closed capacitated network may be formulated as follows:

Find:

$$F^* = \min \left(F = \sum_{(ij)\in G} c_{ij} x_{ij} \right) \qquad \ldots(10)$$

with balance conditions (for nodes):

$$\sum_j x_{ij} - \sum_j x_{ji} = 0, \qquad \text{for each } i \in N \ (i \neq j) \qquad \ldots(11)$$

and constraints (for arcs)

$$l_{ij} \leq x_{ij} \leq u_{ij} \qquad (ij) \in G \qquad \ldots(12)$$

where N and G are sets of all nodes and all arcs in a network, respectively.

If assumption made is that all unit costs, flows and limits on capacities are integer numbers, and flows and limits are non-negative, the feasible flow through a network is a set of flows satisfying relations (11) and (12). The optimal flow in the network is set of flows, which additionally satisfies relation (10).

4.4. Solving procedure via NetLP

To obtain network linear program for given TP there is no need for writing any equation of the model (10)-(12). Rather, it is only a matter of understanding the problem and following some rules and definitions for creating appropriate oriented closed capacitated network while anticipating model (10)-(12). Remaining the same, original network given on Fig. 2 has only to be supplemented by some artificial nodes and links creating closed network, which may be denoted as artificial network, Fig. 4.

From NetLP standpoint, original network on Fig. 4a is not closed and does not bring information on node characteristics (source and/or sink capacities). Closing the network may easily be performed by adding three artificial nodes:

- Source node
- Sink node
- Balance node

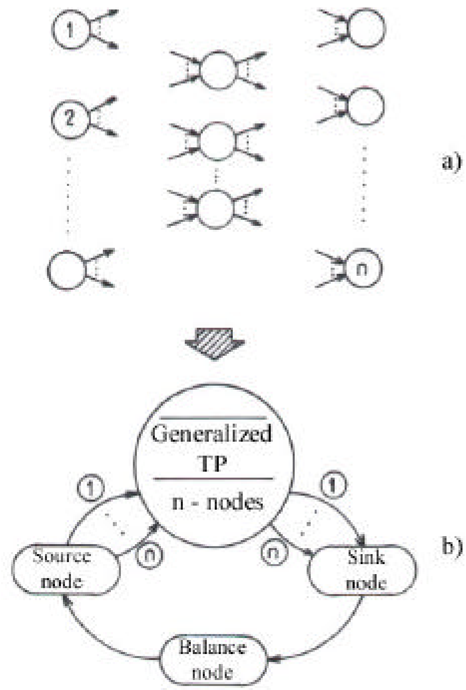

Fig. 4 NetLP formulation of generalized TP

The first one represent common source for all nodes where sources of resource occur. The other one is common sink for all nodes where resource sinks. The last one serves for balancing all amounts of resource within the network. The artificial nodes have to be connected mutually and with original nodes by additional artificial arcs as follows:

- The arcs leaving artificial source node lead to all nodes where source is defined. Upper and lower flow limit for particular arc are equal to each other and equal to 'source capacity' of a node to which that arc leads. In this way through that arc will flow exactly that volume of water specified to be available at source node for further distribution through the network. Unit costs for those arcs may be specified as zero in order not to influence the objective function.

- To the artificial sink node lead a set of artificial arcs originating at nodes where sinks are specified. For given arc the upper flow limit should be equal to specified sink capacity at related node from which it originates. The lower limit should be specified as zero. Thus resource may flow through the link with volume ranging from zero to maximum capacity (demand). Unit costs for these arcs should be also specified as zero for the same reason as above.

- To the artificial balance node arrives only one arc from the artificial sink node with the minimum capacity equal to zero and maximum capacity equal to the sum of capacities of all sink nodes. Unit cost for this arc is zero as before. From the balance node only one artificial arc leaves to the artificial source node. Its maximum and minimum capacity is equal to a sum of source capacities at original nodes. Unit cost flow is equal to zero.

Fig. 4b illustrates the principle of adding artificial nodes and arcs creating thus the final network consistent with model (10)-(12) to be solved.

With such a statement of a problem, optimal solution shall be represented by set of flows through arcs for which overall (total) cost of flow on complete network is the minimal one. It is intuitively obvious that any algorithm applied for solving such a problem will 'seek' for distribution situation within a network for which the highest flows occur in the arcs with the lowest unit costs. Practical reasons indicate that favorite flows may be established by defining negative unit costs for specific arcs. Underlying logic is quite simple: increasing flow in the arc with negative cost decreases related part of the objective function (10). As far original network is concerned, the only difference between StdLP and NetLP formulation of the problem is that in latter case for all arcs upper and lower capacity limits have to be defined. By the analogy to StdLP, here for lower limits of flow zeros may be specified, and for upper limits some unique high value may be adopted.

The NetLP represented by Fig. 4b and relations (10)-(12) is completely equivalent to the StdLP represented by Fig. 2 and relations (16)-(18). Those are starting points for applying Simplex and Out-of-Kilter algorithm.

As far Out-of-Kilter is concerned, recent investigations indicate that this famous Ford - Fulkerson's minimal cost flow network procedure is recognized as very efficient one for solving TPs. Theoretical background and application characteristics of that algorithm are well known and may be found in pertinent literature [1,3].

Few final remarks are given to help better understanding of a network modeling strategy:

1. The number of artificial nodes on Fig. 4b may be reduced by avoiding 'Source' and 'Sink' nodes. The 'Balance' node should remain with all artificial arcs being connected to it in the same manner as before. Of course, two artificial arcs connecting Balance node with Source and Sink nodes no more exist. However, previous transformation is more general providing an easier handling of generalized (open) transportation problems.

2. The unit costs on artificial arcs are equaled to 0 to suppress influence of their flows on the objective function. However, this is not necessary condition. Unit costs may be different from zero and even negative. The only thing left to analyst is to extract artificial part of flow costs from the objective function (10). Significant advantage of network LP modeling over standard LP is just in fact that in generalized TP deficits or sufficits of resource occur. By suitable modifications of unit costs on artificial part of a network, priorities on flows may be defined and general strategy on handling deficits and sufficits may easily be implemented

3. The capacities on arcs provide flexible variation of 'bottlenecks' and 'enlargements' of flow directions, both in original and final network. This is usual situation in both classical and generalized TPs.

5. THE NetLP OR StdLP

General remarks given in previous chapter on issues concerning NetLP as alternative to StdLP solving procedure, represent necessary framework for discussion on principal characteristics of both approaches. Assuming that user 'equally knows' theoretical background of both approaches and has in hand appropriate software, the selection of a method for solving linear optimization problems of transportation or transshipment type is primarily subjective choice. To eliminate such subjectivity the following discussion intents to put a stress on consequences whatever decision is made.

1. Physical systems, processes and phenomena sometimes are just networks or 'alike' them. In such cases NetLP is generally better to select, particularly if it is necessary to analyze alternative statements of a problem [4], to simulate dynamical situations, etc. It is not rare situation that NetLP is used within mixed optimization/simulation (or vice versa) models for planning or operating large-scale systems [5,6]. When time needed for solving problem is not critical, like in single static optimization cases, StdLP and NetLP may be accepted as nearly equal.

2. The constraint and condition relations in StdLP formulation usually imply inclusion of many slack and artificial variables to convert original problem into standard form. Similarly, in NetLP formulations artificial nodes and arcs are necessary to add. In both cases the objective function expands. However, in latter case the expansion of initial problem is easier to handle. For example, constraints of the >, < or = type in StdLP imply rules for including new variables; in NetLP it is enough only to vary upper or lower arc capacities in respective arcs; constraint = requests only to equal upper and lower capacity of the arc in NetLP, and not inclusion of artificial variable like in StdLP.

3. Reformulating the original problem is generally more complicated when StdLP is used. For example, adding new nodes or new transport lines usually means writing new constraint relations and/or renumeration of variables. In NetLP this is achieved by simple exchanging one arc with two new arcs;

arc renumeration is not necessary and just addition of new arcs to already existing set of arcs should be done.

4. The Simplex algorithm is not restricted to only integer linear problems, and Out-of-Kilter is. In later case, problem may be eliminated by use of appropriate integer multiplier to put original problem into integer mode. This means that whole problem containing decimal numbers should be multiplied with some unique integer (10,100,...) to make capacities and unit costs being integer values. When solution is obtained, it should be divided by same number to get the exact solution. This 'game' is in consistence with linear structure of a problem, but obviously brings some complication in input/output data manipulations.

5. The Out-of-Kilter is by its nature network algorithm. As sufficiently general, it may be directly applied to wide range of network or network-oriented problems. Obviously, any transportation problem is at least 'network alike'. Simplex is significantly less suitable for attacking such problems and some of it's special types should eventually be used, still with complications.

6. NetLP seems to be better choice when given TP problem is of large dimension. For example, it is easier to draw new nodes and links than to write new constraint relations and achieve balancing at nodes. In practice, this appears to be significant advantage of NetLP over StdLP.

7. Software packages for StdLP with referenced documentation (Programmers and Users Guides) for years have been available on mainframes at relatively small cost or free of charge. Today, they may be found implemented at PCs too, although primarily as non-standardized packages (except LINDO). On the contrary, network software has always been dominantly commercial. The interesting matter is that in literature it is almost impossible to find source code of any network algorithm. Recent NetLP experience indicates that its practical implementation raises need for expert consulting. Even this situation seems to be normal today and in consistence with modern tendencies of wider use of networks in science and technology, this is still significant disadvantage of NetLP.

6. CONCLUSIONS

The final conclusion concerning superiority of any of two presented approaches to TP may not be given for many reasons. The major one is that both are popular in theory and practice. StdLP has a longer tradition and NetLP is relatively new one. The education of systems analysts, operational researchers and other professionals, particularly in undeveloped and middle-developed countries, still brings the traditional approaches to modern problems. Traditional methods of education are still prevailing, giving as a consequence situation that most of transportation or transshipment problems have been solved by the Simplex method, even manually. Network modeling could find place only in obviously likely network problems like 'shortest (longest) route problem', 'assignment problem', 'critical path problem (CPM and PERT)', 'maximum flow problem', 'maximum capacity problem', etc.

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